EX : - 8.4

Prove that

(i) $(\text{cosec}\,\theta - \cot\theta)^2 = \dfrac{1-\cos\theta}{1+\cos\theta}$

(ii) $\dfrac{\cos A}{1+\sin A} + \dfrac{1+\sin A}{\cos A} = 2\sec A$

(iii) $\dfrac{\tan\theta}{1-\cot\theta} + \dfrac{\cot\theta}{1-\tan\theta} = 1 + \sec\theta\,\text{cosec}\,\theta$

(iv) $\dfrac{1+\sec A}{\sec A} = \dfrac{\sin^2 A}{1-\cos A}$

(v) $\dfrac{\cos A - \sin A + 1}{\cos A + \sin A - 1} = \text{cosec}\,A + \cot A$, using the identity $\text{cosec}^2 A = 1 + \cot^2 A$.

(vi) $\sqrt{\dfrac{1+\sin A}{1-\sin A}} = \sec A + \tan A$

(vii) $\dfrac{\sin\theta - 2\sin^3\theta}{2\cos^3\theta - \cos\theta} = \tan\theta$

(viii) $(\sin A + \text{cosec}\,A)^2 + (\cos A + \sec A)^2 = 7 + \tan^2 A + \cot^2 A$

(ix) $(\text{cosec}\,A - \sin A)(\sec A - \cos A) = \dfrac{1}{\tan A + \cot A}$

(x) $\left(\dfrac{1+\tan^2 A}{1+\cot^2 A}\right) = \left(\dfrac{1-\tan A}{1-\cot A}\right)^2 = \tan^2 A$

Sol. (i) L.H.S. = $(\text{cosec}\,\theta - \cot\theta)^2$

$$= \left(\frac{1}{\sin\theta} - \frac{\cos\theta}{\sin\theta}\right)^2 = \frac{(1-\cos\theta)^2}{\sin^2\theta}$$

$$= \frac{(1-\cos\theta)}{1-\cos^2\theta} \qquad [\because \sin 2\,\theta = 1 - \cos 2\,\theta]$$

$$= \frac{(1-\cos\theta)\times(1-\cos\theta)}{(1-\cos\theta)\times(1+\cos\theta)} \qquad [\because\ 1 - \cos^2\theta = (1-\cos\theta)(1+\cos\theta)]$$

$$= \frac{1-\cos\theta}{1+\cos^{q}} = \text{R.H.S.}$$

(ii) L.H.S$= \frac{\cos A}{1+\sin A} + \frac{1+\sin A}{\cos A} = \frac{\cos^2 A + (1+\sin A)^2}{(1+\sin A)\cos A}$

$$= \frac{\cos 2A + 1 + \sin^2 A + 2\sin A}{(1+\sin A)\cos A} = \frac{(\cos^2 A + \sin^2 A) + 1 + 2\sin A}{(1+\sin A)\cos A}$$

$$= \frac{1+1+2\sin A}{(1+\sin A)\cos A} \qquad [\because \cos^2 A + \sin^2 A = 1]$$

$$= \frac{2+2\sin A}{(1+\sin A)\cos A} = \frac{2(1+\sin A)}{\cos A\,(1+\sin A)}$$

$$= \frac{2}{\cos A} = 2\sec A = \text{R.H.S.} \qquad \left[\because \frac{1}{\cos A} = \sec A\right]$$